TOPOLOGY FINAL EXAMINATION

You can quote any result proved in class (unless you are being asked to prove it). Total marks: 60

(1) (a) Define the notions of sequential compactness and limit point compactness. (2+2 = 4 marks)
    (b) Prove that the Lebesgue covering lemma holds for a sequentially compact metric space. (6 marks)

(2) (a) Define the notions of a second countable, Lindelof and separable topological space. (2+2+2 = 6 marks)
    (b) Prove or disprove: $\mathbb{R}_l$ (the reals with the lower limit topology) is second countable. (6 marks)

(3) (a) Define the notions of a normal, regular, and completely regular topological space. (2+2+2=6 marks)
    (b) State Urysohn's Lemma, Urysohn's Metrization Theorem and the Tietze Extension Theorem. (2+2+2=6 marks)
    (c) Give an example (with details) of a topological space for which Urysohn's lemma does not hold. (6 marks)

(4) (a) Prove or disprove: a locally compact Hausdorff space is completely regular. (6 marks)
    (b) Prove that the one point compactification of the natural numbers $\mathbb{N}$ with the discrete topology is homeomorphic to the subspace $\{\frac{1}{n} | n \in \mathbb{N}\} \cup \{0\}$ of $\mathbb{R}$. (6 marks)

(5) (a) State Tychonoff's Theorem. (2 marks)
    (b) Prove that the countable product (with the product topology), $[0,1]^\omega$, of the closed interval $[0,1]$, is sequentially compact. (6 marks)

Date: April 21, 2015.